

Classical Mechanics
ISI B.Math
Midterm Exam : September 10, 2026

Total Marks: 60

Time : 2.5 hours

Answer all questions:

1. (Marks = 5 + 5 + 5 = 15)

a) A particle with polar coordinates r, θ which are functions of time t is moving in a plane. The velocity and acceleration of the particle can be written in plane polar coordinates as $\mathbf{v} = v_r \hat{\mathbf{r}} + v_\theta \hat{\boldsymbol{\theta}}$ and $\mathbf{a} = a_r \hat{\mathbf{r}} + a_\theta \hat{\boldsymbol{\theta}}$. Find $v_r, v_\theta, a_r, a_\theta$ as functions of $(r, \theta, \dot{r}, \dot{\theta}, \ddot{r}, \ddot{\theta})$

b) A racing car moves on a circular track of radius b . The car starts from rest and its *speed* increases at a constant rate α . Find the angle between the velocity and acceleration vectors at time t .

c) Is the work done by the force $\mathbf{F} = \frac{a \sin t}{r^2} \hat{\mathbf{r}}$ on a particle of mass m in moving in between two fixed coordinate locations independent of path ? a is a constant. Is the total mechanical energy conserved in the process ? Explain.

2. (Marks = 2 + 4 + 3 + 6 = 15)

(a) A particle moves under the influence of the potential $U(x) = A/x^2 - B/x$ where $A, B > 0$.

(i) Make a rough sketch of the potential as a function of x

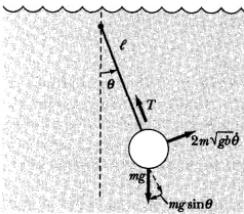
(ii) Show that the motion of the particle consists of either (i) periodic oscillation between two turning points or (ii) an unbounded motion with one turning point, depending upon the value of total energy.

iii) Find the frequency of small oscillations about the stable equilibrium point.

(b) Consider a particle of mass m whose motion starts from rest in a constant gravitational field g . If a resisting force proportional to the square of the velocity (i.e, kmv^2) is encountered, show that the distance s the particle falls in accelerating from v_0 to v_1 is given by

$$s(v_0 \rightarrow v_1) = \frac{1}{2k} \ln \left[\frac{g - kv_0^2}{g - kv_1^2} \right]$$

3. (Marks = 8 + 4 + 3 = 15)



Consider a pendulum of length l and a bob of mass m at its end. The pendulum makes an angle θ with the vertical. The pendulum is moving through oil with θ decreasing. The massive bob

undergoes small oscillations, but the oil retards the bob's motion with a resistive force proportional to the speed with $F_{res} = 2m\sqrt{\frac{g}{l}}(l\dot{\theta})$. The bob is initially pulled back at $t = 0$ with $\theta = \alpha$ and $\dot{\theta} = 0$.

(a) Find the angular displacement θ and velocity $\dot{\theta}$ as a function of time. Is the total mechanical energy conserved? Explain.

(b) Now the pendulum and bob are removed from the oil and the pendulum is allowed to oscillate with the same initial conditions. Find $\theta(t)$. Make a rough plot of θ versus $\dot{\theta}$ under these conditions.

4. (Marks = 3 + 3 + 6 + 3 = 15)

(a) Do the Lagrangians $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ and $L' = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + b\dot{x}t^2 + 2bxt$, where b is a constant, represent the same physical system? Justify your answer.

(b) Write down the Lagrangian for a free particle in cylindrical coordinates. Identify the cyclic coordinate and the corresponding conserved momentum.

(c) A simple pendulum of length b and a bob with mass m is attached to a massless support moving vertically upward with constant acceleration a . Determine (i) the equations of motion and (ii) the period for small oscillations.